MA431: Homework 5

Required Reading:

- MacCleur, Sections 4.3 and 4.4.
- Gelfand & Fomin, Sections 2.12, 3.13(last two pages), 4.16, and 4.17.

Due May 4th at the start of class.

1. MacCluer: 4.26.
2. MacCluer: 4.27.
3. Gelfand & Fomin: 4.3 (Don't compute the Poisson brackets.)
4. One reason for reformulating problems in terms of canonical variables is that many interesting results become easier to prove. Show that if (p_0, q_0) is a critical point of the Hamiltonian system

$$\dot{p} = -\frac{\partial H}{\partial q}, \quad \dot{q} = \frac{\partial H}{\partial p}$$

then this critical point can not be linearly asymptotically stable. What does this mean physically and why does that make sense? (Hint: show that the eigenvalues of the Jacobian for the linearized system can't both have negative real parts. Another hint: chapter 11 in your old DE textbook.)

5. Gelfand & Fomin: 2.18.
6. Gelfand & Fomin: 2.22. (Hint: the free boundary condition is *very* helpful in reducing the EL equations to something simple.)
7. In MA 336: Boundary Value Problems, much attention is paid to the eigenvalue problem

$$y''(x) = \lambda y(x) \qquad y(0) = 0 \qquad y(L) = 0$$

It can be shown that this problem has nontrivial solutions only for $\lambda = -(n\pi/L)^2$ where n is any positive integer, and the solutions are $y_n(x) = c_n \sin(n\pi x/L)$, for arbitrary constants c_n.

(a) Consider rewriting the ODE in the form $\lambda y - \frac{d}{dx} y'(x) = 0$. Use this perspective to formulate a relevant variational problem with functional $J(y)$ and an isoperimetric constraint that normalizes the root mean square to equal 1.

(b) Impose the constraint from part a) to compute the constants c_n, then compute $J(y_n)$. Compare $J(y_1)$ to $J(ax(L-x))$ for the appropriate value of a.

Additional Practice
MacCluer: 4.25, 4.28, 4.30/31
Gelfand & Fomin: 4.1, 4.2