MA431: Homework 4

Required Reading

- MacCleur: sections 3.7 and 4.3.
- Gelfand & Fomin: sections 1.5, 2.10, 2.11, 4.21, 7.35, 7.36.

Due April 18th at the start of class.

1. MacCleur: 3.12/4.13. Don't simply plug numbers into the results from class. Show the full derivation. Call the positions of the masses (x, y) and (X, Y), write the Lagrangian, then transform it to a polar coordinate system.

2. MacCleur: 3.21

3. Mac Cleur: 3.25. Hint: Imagine a cylindrical coordinate system centered around the y axis, then something rotating about the y axis with angular speed ω could be described by $x(t) = R(t)\cos(\omega t + \phi)$ and $z(t) = R(t)\sin(\omega t + \phi)$, where $R(t)$ is related to $y(t)$.

4. Gelfand & Fomin: 2.2.

5. Gelfand & Fomin: 7.1.

6. For a Lagrangian $L(t, x, \dot{x})$, we derived a special form of the Euler-Lagrange equation when L didn't explicitly depend on t.

   (a) Show that if $L(t, x, y, \dot{x}, \dot{y})$ doesn't explicitly depend on t then the EL equations imply $L - \dot{x}L_{\dot{x}} - \dot{y}L_{\dot{y}}$ is constant.

   (b) If $L(x, y, u, u_x, u_y)$ doesn't explicitly depend on (x, y), then it is not necessarily true that the EL equation implies $L - u_x L_{u_x} - u_y L_{u_y}$ is constant. Consider the examples $L = u_x u_y$ and $L = u_x + u_y$.

7. In class we studied the Lagrangian for an elastic membrane, whose potential energy density was proportional to the square of the gradient's magnitude, *i.e.*, $dU = \frac{1}{2}k\|\nabla u\|^2 dA$. Consider now a linearly elastic beam in 1D, whose potential energy density is proportional to the square of it's linearized curvature, *i.e.*, $dU = \frac{1}{2}k(u'')^2 dx$. State an appropriate variational problem and find the Euler-Lagrange equation for a linearly elastic beam. Assume all material parameters are constants. Also assume that u and u' are fixed on the boundary.

Additional Practice
MacCleur: 3.20
Gelfand & Fomin: 1.21, 7.2