MA431: Homework 2

Required Reading

- MacCluer, sections 4.1, 4.2, and 4.5.
- Gelfand and Fomin, sections 1.3-1.4.

Due March 24 at the start of class.

1. MacCleur: Problem 3.3
2. MacCleur: Problem 3.30, Hint: Green's Theorem from vector calculus can be helpful.
3. MacCleur: Problems 4.6
4. MacCleur: Problems 4.20, The author asks for the minimizer. Because we do not yet have tests for sufficiency, we can only find extremals and hope that they provide minima. You may find solving the EL equations quite aggravating. Can you guess a simple solution and then verify that it solves the EL equation?
5. MacCleur: Problems 4.38
6. Gelfand & Fomin: Problem 1.18
7. Consider a mass attached to an ideal Hookean spring with no dissipation subjected to known external force $f(t)$. Suppose the mass is initially at rest and displaced 1 unit from equilibrium. Assume that the natural vibrational frequency is 1 radian per unit time.
   (a) State, but do not solve, an initial value problem describing the displacement of the mass, $y(t)$.
   (b) A second spring/mass system has natural frequency 2 and is unforced. Its initial displacement and velocity are identical to that of the first system. You've decided that the forcing of the first system should be chosen so that the the dynamics are as similar as possible to that of the second system. Write a functional J which, in a sense, measures how *close* the displacement of the first mass is to the displacement of the second mass.
   (c) Based on your answer to part b), state a variational problem for $f(t)$. Include all constraints and boundary/initial conditions in your formulation. Explain why this problem isn't in a form that you could apply our current methods to.

Additional Practice
MacCleur: 4.18, 4.19, 4.46. Gelfand & Fomin: 1.22.