MA431: Homework 1

Due March 14 at the start of class.

For problems marked with * you are encouraged to use software to assist you.

1. MacCleur: Problem 1.19 and read sections 1.1-1.5
2. MacCleur: Problem 2.12 and read sections 2.1, 2.4
3. MacCleur: Problems 3.4, 3.6, and read sections 3.1-3.6
4. Gelfand & Fomin: Problem 1.15bd and read sections 1.1-1.2
5. * In class we considered the following problem

$$J(y) = \int_0^1 (y^2 + (y')^2)dx \qquad\qquad y(0) = 0 \qquad\qquad y(1) = 5$$

where the goal was to find $y \in \mathcal{C}^1$ which minimizes $J(y)$. In class we proved that the following condition was necessary for a weak minimum

$$\int_0^1 (hy + h'y')dx = 0$$

for all $h \in \mathcal{C}^1$ with $h(0) = h(1) = 0$. It's possible to show that minimizing solutions have the form $y(x) = ae^x + be^{-x}$.

(a) Verify that y satisfies the necessary condition above. Hint: Integration by parts.

(b) Compute a and b.

(c) Consider $z(x) = C_2x^2 + C_1x + C_0$. Choose the coefficients to satisfy the boundary conditions and minimize $J(z)$.

(d) Compare $J(z)$ to the minimum value $J(y)$. Plot y and z on the same axis. What is important about this result?

Additional Practice

MacCleur: 1.4, 1.8, 1.18, 2.1, 2.2, and 2.11. Gelfand & Fomin: 1.14 and 1.15ace.