

MA212: Extra Assignment

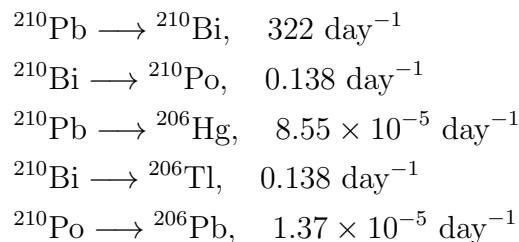
Any problems marked with * require the use of maple. All other problems are to be done by hand.

1. Suppose a water purification system is composed of three 1000 liter holding ponds, each of which pumps liquid to and from the other two.

- Fluid from pond 1 is being pumped into pond 2 at rate 1 L/s and into pond 3 at rate 2 L/s.
- Fluid from pond 2 is being pumped into pond 1 at rate 2 L/s and into pond 3 at rate 1 L/s.
- Fluid from pond 3 is being pumped into pond 1 at rate 1 L/s and into pond 2 at rate 2 L/s

Ponds 2 and 3 initially contain pure water. Pond 1 initially has a contaminant concentration of 5g/L.

- (a) Setup a matrix ODE initial value problem describing this system.
 - (b) * Solve the problem you wrote in part a).
 - (c) Does the contaminant level in the ponds oscillate?
 - (d) What happens to the contaminant level in the long term?
2. Radioactive ^{210}Pb undergoes beta decay to become ^{210}Bi which again undergoes beta decay to become ^{210}Po . These three all undergo alpha decay to become ^{206}Hg , ^{206}Tl , and ^{206}Pb , respectively. ^{206}Pb is stable and doesn't undergo any additional decay. Decay proceeds at a rate proportional to the amount present, with the rate constants for these decays given below. Ignore all other possible decays and assume that the only element present initially is 1 gram of unstable ^{210}Pb .



- (a) Write a matrix ODE initial value problem for the amounts of each element.
- (b) * Solve the problem you wrote in part a).
- (c) How much time will be required for half of the initial unstable Pb to be converted into stable Pb?

3. Consider the ODE system

$$\begin{aligned}x'' - y + x &= 1 \\y'' + y' + x' - y &= 0\end{aligned}$$

- (a) Convert this to a system of four first order ODE.
 - (b) * Find the general solution.
 - (c) Write the expression for $y(t)$.
4. Two identical warm objects are placed into a cold room. The objects are touching each other. The objects give off heat, causing the air in the room to become warmer.

$$\begin{aligned}x' &= k(z - x) + q(y - x) \\y' &= k(z - y) + q(x - y) \\z' &= -q(z - x) - q(z - y)\end{aligned}$$

- (a) Explain why the model above describes this situation. Be explicit about the meaning of all variables and parameters.
- (b) Write this as a matrix ODE system and compute the general solution.
- (c) Are there critical points? If so, what are their stability properties?
- (d) Based on your answer to part b), what do you think happens in the long term? Support your answer with intuition about the physical problem.