MA212: Assignment #9

Required Reading:

- Sections 12.2-12.3

Any problems marked with * require the use of maple. All other problems are to be done by hand. Any problems marked with # can be submitted for review by the grader.

1. Textbook §12.2: 5, 9, 12#, 17, 18#
2. Textbook §12.3: 11, 12#, 13, 20#, 25, 28#, 39, 40#, 52*#
3. Let $f(x) = x$ on $[-\pi, 0]$
   (a) Sketch the even periodic extension of $f(x)$ to the interval $[-2\pi, 2\pi]$. Call this $f_e(x)$.
   (b) Find a Fourier series expansion for $f_e(x)$ on $[-\pi, \pi]$.
   (c) Sketch the odd periodic extension of $f(x)$ to the interval $[-2\pi, 2\pi]$. Call this $f_o(x)$.
   (d) Find a Fourier series expansion for $f_o(x)$ on $[-\pi, \pi]$.
4. #* Consider the following boundary value problem.

$$y''(x) + y(x) = x, \qquad y'(0) = 0, \qquad y'(2) = 0$$

   Notice that the boundary conditions match those of a Fourier cosine series on $[0, 2]$.
   (a) Sketch the even periodic extension of x, then expand x in a Fourier cosine series.
   (b) Plug $y = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos(n\pi x/2)$ into the ODE, together with your expansion from part a).
   (c) Solve for the unknown a_n. Then plot the first 10 terms in the series. Compare to the exact solution

$$y = x - 2\frac{\sin(x)}{\sin(2)}.$$