MA212: Assignment #8

Required Reading:

- Seciton 11.3-11.4

Any problems marked with * require the use of maple. All other problems are to be done by hand. Any problems marked with $^\#$ can be submitted for review by the grader.

1. Textbook §11.3: $28^\#$, 29, $38^{*\#}$, Note for 38, linear analysis will show you that a critical point is a center. Because this doesn't allow conclusions to be drawn about the nonlinear system, examine the full nonlinear system by drawing a phase plane. Be sure to include at least 4 trajectories, chosen carefully so that they reveal the nature of the critical point's stability/instability.

2. Textbook §11.4 #'s 9, $16^\#$, 21, $22^{*\#}$

3. In Florida swamps, Burmese pythons and alligators are top predators with plenty of food. Even so, they sometimes attack and kill each other. For simplicity we'll assume that the two predators have identical population growth characteristics and that neither is more likely to be the winner of their confrontations. After some rescaling, we now have a model with only one parameter.

$$P' = P(1 - P) - kAP$$
$$A' = A(1 - A) - kAP$$

   The parameter k describes the extent to which these two predators kill each other. A large value of k implies a large amount of deadly confrontation. Assume that $k > 0$ and $k \neq 1$.

   (a) This system has four critical points. Find them.

   (b) Which critical point describes the case where the predators coexist? Determine the stability and type of this critical point for all values of $k > 0$ with $k \neq 1$.

   (c) * Set $k = 1/2$. Can the predators coexist in a stable equilibrium? Use eigenvalues and a phase plane plot to support your answer.

   (d) * Set $k = 2$. Can the predators coexist in a stable equilibrium? Use eigenvalues and a phase plane plot to support your answer.

   (e) Give an intuitive explanation for the difference you observed in parts c) and d).

4. $^{\#*}$ The phase plane methods you've learned apply to higher dimensions also. Consider the following third order ODE

$$x''' + 3x'' + 7x' + x^5 = 1$$

   Convert this to a system, show that it has only one critical point, and that the linearization implies this point is asymptotically stable. How would you name this type of critical point?