MA212: Assignment #6 (Review)

1. Textbook, Chapter 10 review: 3-11, 14 (If eigenvalues are complex, make sure you find real solutions to the ODE.)

2. An electric heating implement is being used to warm coffee. Because of a defect, the temperature of the implement oscillates. As a result, heat is being supplied to the coffee at a rate proportional to $e^t + \sin(3t)$. The coffee itself exchanges heat with the mug it's in, at a rate described by Newton's law of cooling. Let $\{c(t), m(t)\}$ be the temperature of the coffee and mug. Write and solve a linear system of ODE describing this situation. Show that the solution is of the form

$$\begin{bmatrix} c(t) \\ m(t) \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-2kt} + ae^t + b\sin(3t) + c\cos(3t).$$

3. Using methods taught in DE II, solve the following problem.

$$x'' + 2x' + x = e^{-2t} - t, \qquad x(0) = 0, \qquad x'(0) = 1.$$

   Recall that we discussed how to convert this problem into one that our methods work on.

4. A population contains baby mice and mature mice, with populations $\{b(t), m(t)\}$.

   - Baby mice are born at a rate proportional to the number of mature mice.
   - Baby mice reach maturity at a rate proportional to the number of baby mice.
   - Mature mice die at a rate proportional to the number of mature mice.

   Write an ODE system describing this situation. By computing eigenvalues, find a condition for the total mouse population to reach equilibrium, instead of dying off or growing without bound. Solve the system under this condition and identify the transient and steady state part of the solution. Under what condition will the equilibrium population be composed of mostly mature mice?