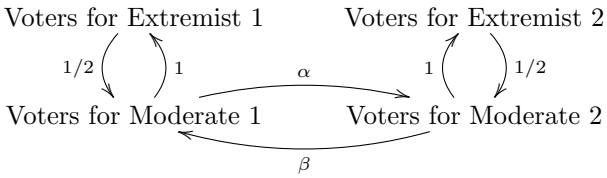
MA212: Assignment # 5

Required reading:

- Textbook sections 10.1, 10.2, 10.4.1

Any problems marked with * require the use of maple. All other problems are to be done by hand. Any problems marked with # can be submitted for review by the grader.

1. Textbook §10.2: 12#, 13, 14#, 15*, 22#, 29, 30#, 36, 38#, 45
2. Textbook: Section 10.4: 1, 4#, 6#, 9
3. * Rework problem 26 of section 10.1 using Maple. Find the solution using Maple's dsolve command. You may find a way to enter matrices into dsolve, or you may want to split the matrix into individual equations.
4. * Rework problem 30 of section 10.2 using Maple. Find the solution using Maple's dsolve command. You may find a way to enter matrices into dsolve, or you may want to split the matrix into individual equations.
5. *# Four candidates are running for president, two from extremist political parties, and two from moderate parties. Suppose, as described in class, voters switch between the candidates over time. The switching rate constants are given in the diagram.



(1)

The rate constants α and β can be increased by a candidate giving an endorsement for the other.

(a) Write an homogeneous matrix differential equation, $X' = AX$, describing the number of voters supporting each candidate. The vector X should be of the form

$$X(t) = \begin{bmatrix} E_1(t) \\ E_2(t) \\ M_1(t) \\ M_2(t) \end{bmatrix}$$

(b) Using only eigenvalues and eigenvectors, describe the vote distribution as $t \to \infty$. Hint: all of your eigenvalues are non-positive numbers.

(c) Using your answer from the previous part, show that it's not possible for either of the moderate candidates to win the election. Then give a mathematical description of how they could adjust α and β to assure that Extremist 1 wins.