MA212: Assignment # 4

Required Reading:

- Sections 2.7 and 10.1

Any problems marked with * require the use of maple. All other problems are to be done by hand. Any problems marked with # can be submitted for review by the grader.

1. Textbook §2.7: 13, 18b#, 21, 26#, 34#, 39ac, 49
2. Textbook §10.1: 3, 11, 12#, 14#, 15, 17, 18#, 21
3. Textbook §10.1: 26#. To prove that a given function is the general solution of an inhomogeneous second order linear system of ODE, the following steps are required
   (a) Write the proposed solution in the form $X(t) = c_1X_1(t) + c_2X_2(t) + X_p(t)$ and identify X_1, X_2, and X_p.
   (b) Show separately that X_1 and X_2 are each solutions of the homogeneous system, and use the definition of a fundamental set to prove that having two solutions is sufficient.
   (c) Show that X_1 and X_2 are linearly independent, and use theorem 10.1.5 to prove that they form the general solution of the homogeneous system. This is known as the complementary solution, X_c.
   (d) Show that X_p a solution to the inhomogeneous system. This is called a particular solution.
   (e) Use theorem 10.1.6 to prove that $X = X_c + X_p$ is the general solution of the inhomogeneous system.
4. One simple model for the growth of a tumor assumes that the growth of new tumor cells proceeds at a rate proportional to their surface area, while the death of existing tumor cells proceeds at a rate proportional to the volume. Define the number of tumor cells to be $N(t)$. We know that if the tumor has a simple shape, then volume is proportional to N. The initial number of tumor cells is N_0.
   (a) For both a sphere and a cube, show that surface area=constant∗volume^n. Based on these two examples, make a conjecture regarding how tumor surface area relates to N.
   (b) Write a differential equation initial value problem for $N(t)$. Your equation will have unknown proportionality constants in it.
   (c) We know that volume has units of length^3. This suggests we introduce a new variable, $N(t) = x(t)^3$. By plugging this into your ODE from part b), show that $x(t)$ solves a linear ODE.
   $$x' + ax = b$$
   where (a, b) are positive constants.
   (d) Solve the ODE in part c), then find an expression for $N(t)$ and fit the initial data. What happens to the tumor volume in the long term, $t \to \infty$?