Worksheet: Intro to Higher Order ODE

1. Consider the Initial value problem $y'' - 3y' + 2y = 10\sin(x)$, $y(0) = 2$, $y'(0) = 0$.

   (a) The homogeneous ODE has two solutions of the form e^{rx}. Find the values of r, confirm the two solutions form a fundamental set, and form the complementary solution.

   (b) The inhomogeneous ODE has a particular solution of the form $A\sin(x) + B\cos(x)$. Find A and B and form the general solution of the inhomogeneous ODE.

   (c) Fit the initial data.

2. An object is moving under the influence of gravity near the surface of Earth. There is also a linear drag force on the object due to air resistance. The ball is dropped from a distance D above the ground.

$$my''(t) = -mg - \alpha y'(t) \qquad y(0) = D \qquad y'(0) = 0$$

   Assume $\alpha = m = g = 10$

   (a) The homogeneous ODE has two solutions of the form e^{rt}. Find the values of r, confirm the two solutions form a fundamental set, and form the complementary solution.

   (b) The inhomogeneous ODE has a particular solution of the form At. Find A and form the general solution of the inhomogeneous ODE.

   (c) Fit the initial data.

   (d) For various values of D, How long does it take the ball to hit the ground? Hint: plot D as a function of t. How does this compare to the case where there is no drag?

3. Consider the ODE $x^3y''' + 2x^2y'' - xy' + y = \ln(x)$. In this problem, you should use Maple to compute determinants.

   (a) Show that the functions $y_1(x) = x$, $y_2(x) = x\ln(x)$, $y_3(x) = 1/x$ form a fundamental set of solutions.

   (b) Show that a particular solution is $1 + \ln(x)$.

   (c) Form the general solution.