First Order ODE

1. A metal object is initially at temperature 100 units. It's placed into large bath of water, kept at a constant temperature of 20 units. After five units of time, the new temperature is 80 units.
   (a) Write a differential equation for the rate of change of temperature. Write the initial condition for $T(t)$.
   (b) Solve the initial value problem for $T(t)$ using integration factors, if possible.
   (c) Solve the initial value problem for $T(t)$ using separation of variables, if possible.
   (d) What was the temperature of the metal object after one unit of time?

2. Rose gold is being alloyed by mixing molten copper with molten gold. A 50/50 mixture of copper and gold is being poured into a vessel at rate 10ml/sec. This vessel initially contains 100ml of pure gold. The mixture is drained from the bottom of the vessel at a rate 2ml/sec.
   (a) Write two differential equations, one to describe the rate of change of total volume of liquid and one to describe the rate of change of the volume of gold. Write initial conditions for $V(t)$ and $G(t)$.
   (b) Solve the initial value problem for $V(t)$, and plug this into the ODE for $G(t)$.
   (c) Solve the initial value problem for $G(t)$ using integration factors, if possible.
   (d) Solve the initial value problem for $G(t)$ using separation of variables, if possible.
   (e) At what time will the outpouring mixture be 80/20 in proportion? How much metal will be in the vessel at this time?

3. We've previously discussed the logistic model for the growth of a population, $P' = k\left(1 - \frac{P}{L}\right)P$. Suppose a chemostat containing paramecium is capable of supporting a concentration of up to $L = 100$ paramecium/milliliter. Suppose also that, because of different temperatures during lab operating hours, the growth parameter is $k = 2$ from 8am to 5pm and $k = 1$ from 5pm until 8am. If the concentration at noon is 50 paramecium/ml, what will be the concentration at midnight?