MA211: Assignment # 9

Required Reading.

- Sections 4.5 and 6.1.

Any problems marked with * require the use of maple. All other problems are to be done by hand. Any problems marked with $^\#$ can be submitted for review by the grader.

1. Textbook §4.5: $2^\#$, 3, $12^\#$, 15
2. Textbook §6.1: $2^{*\#}$, 3^*, 9^*, 11^*, $12^{*\#}$, 13ad, $14^\#$. Hint for 11: When trying to find the analytical solution, note that this ODE is neither linear nor separable. Substitute $z = x + y - 1$ into the ODE and you'll find a simpler ODE for z. Hint for 12: Part a) is asking you to use a high precision numerical solver, *e.g*, Maple's rkf45. Part b) is asking you to compare your answers to what you found in part a).
3. Consider the following initial value problem.

$$y'(x) = \frac{y(x)}{(x-1)^2}, \quad y(0) = 1$$

   Use two steps of Euler's method to estimate $y(3/2)$. The estimate you found is not only inaccurate, but mathematically meaningless. Why?

4. $^{*\#}$Consider the following numerical method for solving $y' = f(x, y)$.

$$\begin{aligned} k &= h * f(x_n + h/2, y_n + k/2) \\ y_{n+1} &= y_n + k \end{aligned}$$

   Notice that computing k usually requires you to solve a nonlinear algebra problem, making this method challenging to use.

   (a) Suppose $f(x, y) = x - y$. Show that it's simple to solve for k in this case.

$$\begin{aligned} k &= \frac{h(x_n - y_n + h/2)}{1 + h/2} \\ y_{n+1} &= y_n + k \end{aligned}$$

   (b) Use this method to approximate $y(1)$ where $y' = x - y$ and $y(0) = 0$. Do so using 10, 100, and 1000 steps. Do you see any benefit to using this more difficult method compared to Euler's method?