MA211: Assignment # 8

Required Reading.

- Sections 4.3-4.4.

Any problems marked with * require the use of maple. All other problems are to be done by hand. Any problems marked with $^\#$ can be submitted for review by the grader.

1. Textbook §4.3: 39, $42^\#$, $44^\#$, 45, $62^\#$, $68^\#$, 69

2. Textbook §4.4: $8^\#$, $10^\#$, 17, $18^\#$, 28, 41, 57, $60^\#$. For each of these problems, ignore the book's specific instructions with regard to which theorems and tables to use. Just use the table distributed in class.

3. The system $y'(t) = y(t)$ exhibits solutions with exponential growth, $y(t) = y_0 e^t$. Suppose this behavior is considered undesirable in your application. In an attempt to remove the growth, you implement an external control mechanism which takes measurements of the system, averages these measurements, and applies a feedback proportional to this average.

$$y'(t) = y(t) + a\frac{1}{t}\int_0^t y(\tau)d\tau, \quad y(0) = 1$$

where a is a real constant.

   (a) Multiply the integro-differential equation by t and transform to find

$$Y'(s) = \frac{s + a}{s(1 - s)}Y(s).$$

   (b) For the special case $a = -1$, solve for $y(t)$ and show that the feedback mechanism eliminates the exponential growth. In fact, it eliminates all changes. Confirm your solution, $y(t)$, by checking the original equation and initial condition.

   (c) For the special case $a = -2$, solve for $y(t)$ and show that the feedback mechanism eliminates the exponential growth but causes a linear growth. Confirm your solution, $y(t)$, by checking the original equation and initial condition.