MA211: Assignment # 4

Required Reading.

- Read §3.3-3.4.

Quiz on Oct. 3rd. Any problems marked with * require the use of maple. All other problems are to be done by hand. Any problems marked with # can be submitted for review by the grader.

1. Textbook §3.3: 1, 4, 12#, 15, 16#, 29, 32, 40#, 46, 48, 69*
2. Textbook §3.4: 1, 2, 6#, 10#, 13, 26#, 27, 28#, 46, 49*
3. When studying the diffusion of particles inside of a cylindrical container, it's possible to convert the resulting differential equations to ones of the form

$$r^2u''(r) + ru'(r) - n^2u(r) = 0$$

where $n = 0, 1, 2, 3, \ldots$ can be any non-negative integer. Here $u(r)$ is the concentration of particles at a distance r from the axis of the cylinder.

(a) Make the change of variable $x = \ln r$ so that the new dependent variable is defined by $v(x) = v(\ln r) = u(r)$. This change of variable will give you a constant coefficient ODE for $v(x)$. Hint: Use the chain rule. For example:

$$u'(r) = \frac{du}{dr} = \frac{dv}{dx}\frac{dx}{dr} = \frac{dv}{dx}\frac{1}{r} = v'(x)e^{-x}$$

(b) Find the general solution, $v(x)$, to the ODE your found in part 3a. There will be cases depending on the value of the integer n. Be sure to describe all cases.

(c) Using your solution, $v(x)$, from part 3b, give the general solution, $u(r)$, of the original ODE. Again, describe all cases.

(d) If your goal is to solve a diffusion problem for the interior of a cylinder, then you expect the concentration to remain finite. What part of your general solution, if any, should be removed to be sure that concentrations remain finite?