MA211: Assignment #2

Required Reading.

- §2.2, 2.3

Exam on Sept. 19th. Any problems marked with * require the use of maple. All other problems are to be done by hand. Any problems marked with # can be submitted for review by the grader.

1. Textbook §2.2 6#, 7, 8#, 17, 30#, 39*, 40* Note for 39&40: Find solutions by hand, plot with Maple.
2. Textbook §2.3 5, 6, 14#, 18, 22#, 25, 26, 52#
3. If an incompressible fluid is flowing steadily, then, at a particular point in the flow, there is a density ρ, a velocity v, and a pressure P. Also, if gravity is considered, then the height h at the point we've chosen is relevant. Bernoulli's principle states that, even as (ρ, v, P, h) change as we move along a streamline to another point in the fluid, the following combination of these variables will remain constant: $P+\rho gh+\frac{1}{2}\rho v^2$, where g is the acceleration of gravity, $9.8m/s^2$.

   Consider a cylindrical bucket with a hole in it. Start at the top of the water level: $P=$atmospheric, $v=0$, $h=y$ and then follow a stream line of water down to the hole: $P=$atmospheric, and $h=0$.

   (a) Use Bernoulli's principle to show that the velocity of fluid exiting the hole is $v=-\sqrt{2gy}$.

   (b) Now suppose the bucket has cross-sectional area A_b and the hole has area A_h. The rate at which the water line lowers is dy/dt. Find the positive constant k, depending on A_b, A_h, and g, so that $dy/dt=-k\sqrt{y}$.

   (c) Suppose the bucket is empty at time $t=1$, *i.e.*, $y(1)=0$. Solve the initial value problem. Does your solution make sense for $t>1$ and $t<1$? Give a physical interpretation to support your answer.

   (d) Notice that $y(t)=0$ is another solution to your intitial value problem, one that isn't a member of the one parameter family you found in part (c). Call the solution you found $f(t)$. Show that the following function also solves the initial value problem.

   $$y(t)=\begin{cases} f(t) & t<1 \\ 0 & t\geq 1 \end{cases}$$

   Recall that for a function to solve a DE it has to have certain properties, which must be checked carefully. Does this solution make sense for $t>1$ and $t<1$? Give a physical interpretation to support your answer.