MA113: Assignment # 9

Required Reading:

- Sections 15.4-15.6

Any problems marked with * require the use of Maple. **All others are to be done by hand.**

To be completed, but not turned in.

- §15.4, #'s 17, 19, 23, 25, 31, 33, 39, 49*
- §15.5, #'s 13, 19, 21, 25, 29, 43, 51*
- §15.6, #'s 1, 11, 19, 23, 29, 31

To be turned in May 15th at the start of class.
Note: It is recommended that you complete the textbook problems before attempting the problems below.

1. *Approximate an apple as a sphere of radius R and uniform density. Now, use an apple corer to punch a cylindrical hole (radius r) through the center of the sphere.
   (a) How will you orient the sphere and which coordinate system is the best choice for computing the volume and moment of inertia about the central axis of the hole?
   (b) Compute the volume.
   (c) Compute the moment of inertia about the central axis of the hole
2. *The plane $x + y + z = 0$ cuts through the ellipsoid $4x^2 + 4y^2 + 4z^2 + 2(xy + yz + xz) = 1$, separating it into two equal pieces. Suppose the density of the ellipsoid is given by $\delta(x, y, z) = z$.
   (a) Plot the plane and ellipsoid on the same set of axes. Use your plot to explain how you would use integrals to compute the mass of the upper piece. How many integrals are required?
   (b) If the coordinate system is rotated, the volume and mass of objects won't change, but the integrals might be easier to compute. Make a plot to show that the substitution
   $$\begin{aligned} x &= \frac{3-\sqrt{3}}{6}a + \frac{3+\sqrt{3}}{6}b + \frac{\sqrt{3}}{3}c \\ y &= \frac{\sqrt{3}}{3}a - \frac{\sqrt{3}}{3}b + \frac{\sqrt{3}}{3}c \\ z &= \frac{-3-\sqrt{3}}{6}a + \frac{-3+\sqrt{3}}{6}b + \frac{\sqrt{3}}{3}c \end{aligned}$$
   will rotate the plane and ellipsoid into a better orientation in the new (a, b, c) coordinate system. How many integrals will be required now?
   (c) Compute the mass of the upper piece.
3. (a) Convert the following integral into polar coordinates, then compute both integrals.
   $$\int_1^2 \int_0^x \frac{xy}{\sqrt{x^2+y^2}}\,dy\,dx$$
   (b) Convert the following integrals into polar coordinates, then compute whichever is easier.
   $$\int_0^1 \int_{\sqrt{y-y^2}}^{1+\sqrt{1+2y-y^2}} \frac{1}{\sqrt{x^2+y^2}}\,dx\,dy + \int_0^{1+\sqrt{2}} \int_1^{1+\sqrt{1+2x-x^2}} \frac{1}{\sqrt{x^2+y^2}}\,dy\,dx$$