MA113: Assignment # 8

Required Reading:

- Sections 15.1-15.3

To be completed but not turned in.
Any problems marked with * require the use of maple.

- §15.1, #'s 3, 7, 13, 19, 23, 33, 37
- §15.2, #'s 9, 11, 17, 23, 27, 41, 43, 51, 59, 93*
- §15.3, #'s 3, 11, 17, 21, 25

To be turned in May 8th at the start of class.

1. A region in the xy-plane, $\mathcal{R}$, is enclosed by the curves $x = 0$, $y = \sqrt{x/3}$, and $y = 1$. Consider the volume below $z = e^{y^3}$ and above $\mathcal{R}$.
   (a) Set up an integral for the volume, where you would integrate in y first.
   (b) Set up an integral for the volume, where you would integrate in x first.
   (c) Decide which of the two expressions for the volume seems easier to compute, and then compute that one only.

2. *The paraboloid $z = x^2 + y^2$ intersects the plane $2x + 2y + z = 2$ trapping a volume in between. Find the volume.

3. A square sheet of metal is being manufactured ($0 < x < L$, $0 < y < L$). Because of the process used to create it, the sheet has been heated to a temperature T_{hot}, while the surrounding air has temperature T_{cool}. The equation $u_t = \kappa(u_{xx} + u_{yy})$ describes the temperature in the sheet as a function of time $u(x, y, t)$. It can be shown (take MA336 to see the solution method) that the temperature in the sheet is given by

$$u(x,y,t) = T_{cool} + (T_{hot} - T_{cool}) \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{4(1-(-1)^2)(1-(-1)^m)}{nm\pi^2} e^{-\frac{\kappa\pi^2}{L^2}(n^2+m^2)t} \sin\frac{n\pi x}{L} \sin\frac{m\pi y}{L}$$

   You can see that, because of the decaying exponentials, the sheet eventually cools back down to the surrounding air temperature, T_{cool}. If we allow enough time to pass, and if the square sheet isn't very large and if the thermal diffusivity, κ, is large (as it often is for metals), then this double infinite sum can be approximated by using only its first term.

$$u(x,y,t) \approx T_{cool} + (T_{hot} - T_{cool}) \frac{16}{\pi^2} e^{-\frac{2\kappa\pi^2}{L^2}t} \sin\frac{\pi x}{L} \sin\frac{\pi y}{L}$$

   (a) Using this last expression, estimate the average temperature in the sheet, $u_{ave}(t)$.
   (b) At what time would this average temperature equal the average of T_{hot} and T_{cool}?