MA113: Assignment # 7

Any problems marked with * require the use of maple. All others are to be completed by hand.

Required Reading

- Textbook, sections 14.7-14.9.

To be completed, but not turned in.

- Textbook, §14.8, # 1, 5, 9, 16, 27, 31b, 39, 43, 49*
- Textbook, §14.9, # 1, 7, 11

To be turned in May 1st at the start of class.

1. Repeat the blood vessel problem from class, but now assume that the branch has a variable radius, that is, r_2 is variable. Assume that the volume of the cylindrical branch is a fixed constant, that is, $\pi r_2^2 L_2 = V$ where V is a fixed constant. When the optimization problem is solved correctly, you will find a single equation which determines the value of θ. That equation will take the form

$$\cos\theta \csc^2\theta = \text{constant}$$

   You do not need to solve the equation, you only need to do the work required to state the equation correctly.

2. * Use the Lagrange multiplier method to find the shortest line segment connecting $y = x^2/2$ to $y = \ln x$. Compute an equation for the line and compute the minimum distance. Confirm your solution by making a plot of both curves and the line segment on the same set of axes. Note: you can probably think of other ways (without Lagrange Multipliers) to solve this problem. You are encouraged to try one of those methods to check you answers.

3. *Suppose you've collected the data points $\{(0,0,0),(1,0,0),(0,1,0),(2,0,-7)\}$ and wish to find a plane of the form $ax + by + cz = 1$ which approximates this data.

   (a) Compute the distance from $(0,0,0)$ to the plane. Call this D_1. Repeat to find D_2, D_3, and D_4.

   (b) Define the function $F(a,b,c) = D_1^2 + \ldots + D_4^2$. Compute the minimum value of F.

   (c) Plot the points and the optimal plane on the same set of axes.

4. Last week you were given some information about a function, $f(x,y)$, and its derivatives at the point $(\pi/2, 0)$ and were asked to use a tangent plane approximation to estimate $f(\pi/3, \pi/12)$. Now that you've learned about Taylor polynomials, return to that problem and use the given information to get a (hopefully) better estimate of $f(\pi/3, \pi/12)$.