MA113: Assignment # 6

Any problems marked with * require the use of maple.

To be completed but not turned in.

- Textbook §14.6 # 3, 19, 23, 31, 49
- Textbook §14.7 #11, 19, 29, 31, 37, 44bd, 49, 57, *73

To be turned in April 24th at the start of class.

1. The following information about the twice differentiable function $f(x, y)$ is known.

$f(\pi/2, 0) = 1,$	$f_x(\pi/2, 0) = 3,$	$f_y(\pi/2, 0) = -2,$
$f_{xx}(\pi/2, 0) = 0$	$f_{xy}(\pi/2, 0) = 1,$	$f_{yy}(\pi/2, 0) = 0,$
$\lvert f(x, y)\rvert \le 10 + x^2y^2,$	$\lvert f_x(x, y)\rvert \le 8 + y^2,$	$\lvert f_y(x, y)\rvert \le x^2$
$\lvert f_{xx}(x, y)\rvert \le 4\lvert\sin(2x + y)\rvert,$	$\lvert f_{xy}(x, y)\rvert \le 1 + 2\lvert\sin(2x + y)\rvert,$	$\lvert f_{yy}(x, y)\rvert \le \lvert\sin(2x + y)\rvert$

   Use this information to precisely estimate $f(\pi/3, \pi/12)$, including a precise bound on the error. Your answer should be simplified and in the form $f(\pi/3, \pi/12) \in [a, b]$.

2. A flat circular metal plate of radius 2, centered at the origin, has been heated on its outer edge according to $T_{edge}(\theta) = 3 + \sin\theta - 2\sin^2\theta$ and allowed to achieve steady state. The temperature distribution inside the plate is found (by students in MA336: Boundary Value Problems) by solving the steady state Heat Equation.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} = 0$$

   The solution they found is

$$T(r, \theta) = 2 + \frac{1}{2}r\sin\theta + \frac{1}{4}r^2(1 - 2\sin^2\theta).$$

   (a) Confirm that the given solution solves the steady state Heat Equation and satisfies the edge temperature condition.

   (b) Find and classify the critical points on the interior of the metal plate. Give exact answers, not numerical approximations. Find the locations and values of absolute (global) maximum and minimum temperatures. Give exact answers, not numerical approximations.

   (c) *Make a 3D polar plot to confirm your answers. (See Maple help file under *plot3d*, *coords*, and *addcoords*. There is one good example in the help file which shows you exactly how to make this type of plot.) Do your answers make sense physically, *i.e.*, where would you have intuitively guessed the max/min temperatures would be? Plotting $T_{edge}(\theta)$ separately might help.