MA113: Assignment # 5

Required Reading.

- Textbook, sections 14.3, 14.4, and 14.5.

Any problems marked with * require the use of maple.

To be completed, but not turned in.

- Textbook §14.3 #25, 29, 49, 61, 82
- Textbook §14.4 #7, 9, 29, 33, 37, 51
- Textbook §14.5 # 9, 13, 21, 27, 30ac, 36

To be turned in April 17th at the start of class.

1. Consider the following function and point.

$$f(x) = \frac{\sin^2 x}{x}, \quad x = 0.$$

For each of the parts below you must show complete work and explanations. A "yes" or "no" answer without supporting proof/evidence/explanation will not be given credit.

(a) Is f defined at the point? Is the following function continuous for all $x \in \mathbb{R}$?

$$f_*(x) = \begin{cases} f(x), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

(b) Is $\frac{df_*}{dx}$ defined at the point? Is the following function continuous for all $x \in \mathbb{R}$?

$$f_*'(x) = \begin{cases} f'(x), & x \neq 0 \\ 1, & x = 0 \end{cases}$$

(c) Using what you've learned in the previous parts, how would you redefine

$$f(x, y) = \frac{\sin^2(y - x)}{y - x}$$

as well as f_x, and f_y so that the resulting function, $f_*(x, y)$ is continuous and differentiable for all $(x, y) \in \mathbb{R}^2$?

2. * The tangent plane approximation gives us a way to study the error of a calculation when there are uncertainties in the values of the independent variables.

$$\text{Error} = f(x, y) - f(x_0, y_0) \approx \frac{\partial f}{\partial x}(x_0, y_0)(x - x_0) + \frac{\partial f}{\partial y}(x_0, y_0)(y - y_0)$$

The wave theory of matter asserts that every moving object has properties which are like that of a wave of a certain length. The formula

$$\lambda = \frac{h\sqrt{1 - v^2/c^2}}{mv}$$

is used to compute this *de Broglie* wavelength. Here m is the object's mass when motionless, v is the object's velocity, c is the speed of light (a constant), and h is Planck's constant. Suppose you measure a Carbon-12 atom moving at 1/10th the speed of light. Because your equipment is old, you know that your velocity measurement could be too low by as much as 10%. You also suspect that this carbon atom may instead be a Carbon-13 isotope.

(a) Use a tangent plane approximation to estimate the worst-case relative error in your calculation of the de Broglie wavelength.

(b) Using some of the work you did in part a), which appears to cause greater errors in your wavelength computation, the error in measuring the velocity or the unknown atom/isotope type?

3. * The following two equations each define a surface in 3D.

$$x^2 + y^2 - z^2 = 1, \qquad (x-3)^2 + (y-1)^2 + (z-1)^2 = 1.$$

(a) Find the equation of a line segment connecting these two surfaces such that the segment is normal to each of the surfaces.

(b) Compute the length of the line segment. Make a good plot of the two surfaces and the line segment on the same set of axes. What do you think is special about the length of this line segment.

4. At the point $(1, -2)$, the gradient of $f(x, y)$ points in the same direction as $\langle 1, 1 \rangle$ while the directional derivative of f in the same direction as $\langle 2, 3 \rangle$ is 5. What is the smallest value of the directional derivative?