MA113: Assignment # 4

Required reading:

- Sections 13.4, 13.5, 14.1, and 14.2.

Any problems marked with * require the use of maple.
Not to be turned in.

1. Textbook §13.4 #1, 9, 15, 29, 33abc*
2. Textbook §13.5 #9, 15, 19, 29*
3. Textbook §14.1 #13, 15, 31, 33, 35, 43, 45, 63, 75* (Note: 13-64 are to be done by hand.)
4. Textbook §14.2 #13, 33, 39b, 41, 51,

To be turned in April 3rd at the start of class.

1. In many textbooks, the following formula is derived.

$$\frac{d\hat{\mathbf{T}}}{ds} = \frac{\mathbf{r}''\|\mathbf{r}'\|^2 - (\mathbf{r}'\cdot\mathbf{r}'')\mathbf{r}'}{\|\mathbf{r}'\|^4}$$

where $\mathbf{r}' = d\mathbf{r}/dt$ and $\mathbf{r}'' = d^2\mathbf{r}/dt^2$. The variable t is not necessarily time. This is the starting point for several formulae involving curvature.

(a) Recall that $v \cdot v = \|v\|^2$. Use the formula above to show

$$\kappa^2 = \frac{\|\mathbf{r}''\|^2\|\mathbf{r}'\|^2 - (\mathbf{r}'\cdot\mathbf{r}'')^2}{\|\mathbf{r}'\|^6}$$

(b) Recall that $u \cdot v = \|u\|\|v\|\cos\theta$. Use the formula derived in 1a) to show

$$\kappa = \frac{\|\mathbf{r}''\|\sin\theta}{\|\mathbf{r}'\|^2} = \frac{\|\mathbf{r}'\times\mathbf{r}''\|}{\|\mathbf{r}'\|^3}$$

where θ is the angle between $\mathbf{r}'$ and $\mathbf{r}''$.

(c) Suppose that $\mathbf{r}$ is restricted to a plane, so that $\mathbf{r}(t) = \langle x(t), y(t), 0\rangle$. Use the formula from 1b) to show

$$\kappa = \frac{|\dot{x}\ddot{y} - \ddot{x}\dot{y}|}{(\dot{x}^2 + \dot{y}^2)^{3/2}}$$

(d) If a curve corresponds to a function in a plane, then $y = f(x)$, and we can represent this curve as $\mathbf{r}(x) = \langle x, f(x), 0\rangle$. Use 1c) to show that

$$\kappa = \frac{|f''|}{(1 + (f')^2)^{3/2}}$$

and then use this to compute the curvature for a parabola, $y = x^2$. How does this differ from the (incorrect) curvature you were taught in Calculus I?

2. A 10-kilogram object is moving along the spiral $x = u \cos u$, $y = u \sin u$ in the direction of increasing u. When it is at the origin, its speed is 3 m/s and it is speeding up at the rate of 7 m/s^2. Compute the following quantities at that point.

   (a) The rate of change of arc length, ds/dt.
   (b) The velocity, $\mathbf{v}$.
   (c) The tangential acceleration, a_T.
   (d) The normal acceleration, a_N.
   (e) The unit tangent, $\hat{\mathbf{T}}$.
   (f) The unit normal, $\hat{\mathbf{N}}$.
   (g) The unit binormal, $\hat{\mathbf{B}}$.
   (h) The curvature, κ.
   (i) The torsion, τ.
   (j) The force on the object.
   (k) The power being applied to the object.

   You may not necessarily want to compute these in the order they are listed. Note that there is enough data given in the problem to compute nearly all of these, but one may be impossible to compute. Which one?