# MA113: Assignment # 3

**Required Reading:**

- Sections 13.1-13.4

Any problems marked with $^*$ require the use of maple.

**Not to be turned in.**

1. Textbook §13.1 #5, 11, 13, 25, 27, 37ae.
2. Textbook §13.2 #5, 7, 21.
3. Textbook §13.3 #1, 3, 5, 9, 17de$^*$.

**To be turned in March 27th at the start of class.**

1. Suppose $t$ is a time variable and two particles have positions described by

$$\mathbf{p}(t) = \langle t - t^2, 2t, 3t^2 \rangle$$
$$\mathbf{q}(t) = \langle 3t + 1, 2t, -6t - 3 \rangle$$

   There is a moment when the particles collide while traveling in the same direction.

   (a) Plot these curves on the same set of axes for a sufficiently large set of $t$ values so that the point of collision is clearly visible.

   (b) At what time do the particles collide, at what point do the particles collide, and what are their speeds and directions at the point of collision?

2. A helix is a curve shaped like a coiled spring. The following equation describes a helix.

$$\mathbf{r}(t) = \langle a\cos(\omega t), a\sin(\omega t), bt \rangle$$

   (a) $^*$ Starting from $t = 0$, plot two revolutions of the helix when $a = b = \omega = 1$.

   (b) Compute the arc length of one revolution of the helix. Your answer should be valid for all positive values of $(a, b, \omega)$.

   (c) When looking at a coiled spring in the shape of a helix, people sometimes think that if they were to press the coil flat, they would get a circle of radius $a$. Is this true? Hint: use part b).

3. A spirograph is a children's drawing toy that creates interesting curves. The equation describing these curves is

$$\mathbf{r}(t) = \langle a\cos(t) + c\cos(bt), a\sin(t) - c\sin(bt) \rangle$$

   where $(a, b, c)$ are positive numbers.
   Pictures of various spirographs can be seen here http://mathworld.wolfram.com/Spirograph.html .

   (a) Use this tool to design your own spirograph curve. http://nathanfriend.io/inspirograph/

   (b) $^*$ Set $(a, b, c) = (1, 1/2, 1/2)$ and make a plot. On the same set of axes, plot the circle $r = 3/2$. Set $(a, b, c) = (1, 1/4, 2)$ and make a plot. On the same set of axes, plot the circle $r = 3$.

(c) You'll see from your plots that these curves can have many points of tangency with a circumscribed circle. For general $(a, b, c)$, find the radius of the circumscribed circle, find the points of tangency, and show that the slopes at these points of tangency are

$$-\cot\left(\frac{2n\pi}{b+1}\right), \quad n \in \mathbb{Z}$$

4. * A particle is restricted to traveling along the curve $r(t) = \langle t\cos(t), t\sin(t)\rangle$ where $t$ is the time parameter. Recall from Newton's Laws that, unless acted upon by outside forces, an object will travel in a straight line with constant speed. Suppose that at some time the particle is suddenly released from the curve and permitted to travel freely. The particle later strikes a detector positioned at $(1, 2)$.

   (a) At what time did the particle strike the detector and at what point on the curve was the particle released from?

   (b) What is the total length of the path traveled by the particle, from $t = 0$ until the time it strikes the detector?

   (c) Plot the path of the particle, from $t = 0$ until the time it strikes the detector.