MA113: Assignment # 2

Not to be turned in.

1. Textbook §12.3 #3, 7, 11, 13, 19.
2. Textbook §12.4 #7, 11, 23, 25, 27.
3. Textbook §12.5 #3, 7, 9, 19, 21, 23, 27, 37, 77

To be turned in March 20th at the start of class.

1. Define the following three functions.

$$x(s,t) = s + t + 1, \qquad y(s,t) = s - t, \qquad z(s,t) = 2s - t.$$

   As s and t vary over all real values, the point (x, y, z) traces out a plane. Find the equation of this plane in standard form.

2. A right handed coordinate system is a set of three unit vectors (the coordinate axes directions), call them $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$ with the property

$$\mathbf{a} \times \mathbf{b} = \mathbf{c}, \qquad \mathbf{b} \times \mathbf{c} = \mathbf{a}, \qquad \mathbf{c} \times \mathbf{a} = \mathbf{b}.$$

   Do the vectors $\{\langle 0, 0, -1\rangle, \langle 1/\sqrt{2}, 1/\sqrt{2}, 0\rangle, \langle 1/\sqrt{2}, -1/\sqrt{2}, 0\rangle\}$ form a right handed coordinate system? If so, sketch the axes of this coordinate system.

3. The projection you learned about in class is used to project a vector onto a single vector. In some applications, it is desirable to project a vector onto a plane. Recall the derivation of the projection formula in class. Instead of projecting u onto v by expressing it as $u = kv + w$, with v orthogonal to w and k a scalar, we'll project u onto the plane formed by two vectors v_1 and v_2.

   (a) Let $u = k_1 v_1 + k_2 v_2 + w$, where v_i are orthogonal to w and k_i are scalars. Use dot products to eliminate w in two different ways, obtaining two equations for the unknown scalars, k_i.

   (b) Use the result of part a) to project $\langle 1, 2, 3\rangle$ into the yz-plane. Make a sketch and use geometric reasoning to confirm that your solution is correct.

4. The line $\mathbf{r}(t) = \langle t, 2t, 1 - t\rangle$ intersects the plane $z = x + 2y$. Show that the line is normal to the plane and compute the point of intersection.