MA113: Assignment # 1

Required reading.

- Read Sections 11.3, 11.4, 12.1-12.3.

Any problems marked with * require the use of maple. All others should be done by hand.

Problems not to be turned in.

1. Textbook, §11.3, #5, 7, 11, 17, 25, 31, 41, 59
2. Textbook, §11.4, #17, 19, 29, 33*, 35*
3. Textbook, §12.1, # 25, 31, 35, 41
4. Textbook, §12.2, #13, 19, 21, 25, 29, 47, 49

Problems to be turned in March 13 at the start of class.

1. For each equation below, convert from polar to cartesian, or from cartesian to polar. Then solve for either $y(x)$ or $r(\theta)$.
   (a) $x^2 + y^2(1 - 1/x^2) = 0$
   (b) $r = \tan\theta \sec\theta$
2. Define $\mathbf{u} = 2\mathbf{i} + 3\mathbf{j}$ and $\mathbf{v} = \langle 1, -1\rangle$.
   (a) Convert $\mathbf{u}$ to a unit vector.
   (b) Compute the magnitude and direction of $\mathbf{v}$.
3. A friend of mine was developing parts for a moving solar array. He was trying to optimize the amount of light exposed to the panels. In the process, he deduced that one of his parts needed to be shaped like the Conchoid of Nicomedes, a famous curve which was known to the ancient Greeks.

$$r = 2 + k \sec\theta$$

   The curve takes various shapes as the parameter k varies. The plot of this function for $k = 2$ is shown.
   (a) *Plot the curve for $k = 2$, reproducing what I've shown. Use the plot range $r \in [0, 10]$ and $\theta \in [0, 2\pi]$, as I have done in the figure. To get the correct figure, you may find the polar plot option *coordinateview* useful, as well as the *display* command for combining multiple plots. I made two plots, with carefully chosen domains and ranges, and then combined them.
   (b) *Plot the curve for k =1 and 4, with the same plot range for r and θ.
   (c) For $k = 1$ you'll see a small loop. Compute the distance from the origin to the most distant part of the loop.

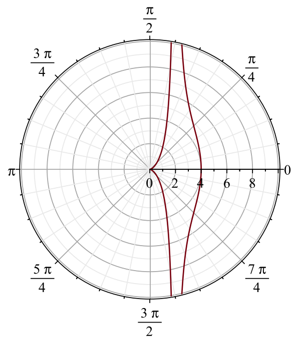

Figure 1: Plot of $r = 2 + 2\sec\theta$.

(d) For $k = 4$, convert the equation to cartesian coordinates. Solve explicitly for $y(x)$ and then simplify. You can see from the plot of the curve that you will find multiple answers.

4. Consider the set of vectors

$$v_k = \cos\left(\frac{2\pi k}{n}\right)\hat{\imath} + \sin\left(\frac{2\pi k}{n}\right)\hat{\jmath}, \quad k = 1, 2, \ldots, n$$

Where n is a fixed positive integer.

(a) For the special case $n = 6$, sketch the vectors $v_1, v_2, \ldots, v_6$.

(b) *Show that, for all positive integer values of n,

$$\sum_{k=1}^{n} v_k = 0\hat{\imath} + 0\hat{\jmath}$$

(c) Using the intuition gained from part a), interpret the result of part b).