Lagrange Multipliers: Optimal Growth of Blood Vessels

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The sketch below shows a small blood vessel of radius r_2 branching from a larger one of radius r_1. As the heart pumps blood, the resistance encountered is proportional to $1/r^4$, where r is the radius of a blood vessel. For efficiency, blood should be moved from the point $(0,0)$ to the point (c,b) with minimal resistance. We'll ignore the fluid resistance which occurs when blood transfers from the larger vessel to the smaller one.

$$R = a\left(\frac{L_1}{r_1^4} + \frac{L_2}{r_2^4}\right) \qquad \text{(Total Hydraulic Resistance)}$$

$$c = L_1 + L_2\cos\theta \qquad \text{(fixed x location of the end of the branch)}$$

$$b = L_2\sin\theta \qquad \text{(fixed y location of the end of the branch)}$$

(a, b, c, r_1, r_2) are known fixed constants. (L_1, L_2, θ) are variables. Use Lagrange multipliers to determine (L_1, L_2, θ) which minimize the resistance. Show and explain your work.