

MA113: Velocity and Acceleration

1. A man with a mass of 70 kilograms is running in the clockwise direction around a circular track whose equation is $x^2 + y^2 = 25$. When he gets to the point (3,4), he is running 5 m/s and he is speeding up at the rate of 10 m/s². What is his acceleration vector at that moment?
2. In the xy-plane where distances are measured in meters, a 40 kg object is moving from left to right on $y = e^{-x}$. At $x = 0$ its speed is 2 m/s, and it is speeding up at the rate of $\sqrt{8}$ m/s². What is the total force on it at that point? What is the magnitude and angle of inclination of this force?
3. An object is traveling, in an xy-plane where distances are measured in centimeters, counterclockwise around the ellipse $C : x = 6 \cos(\theta), y = 8 \sin(\theta)$. When it is at $(-6, 0)$, its acceleration vector is $6i + 4j$ cm/s². What is its speed and at what rate is it speeding up or slowing down at that point?
4. A 10-kilogram ball is moving, in the direction of increasing α , on the spiral $x = \alpha \cos(\alpha), y = \alpha \sin(\alpha)$. When it is at the origin, it is traveling 3 m/s and is speeding up at the rate of 7 m/s². What is the total force on it at that point?
5. At a point P in an xy-plane where distances are measured in meters, the unit tangent vector to an object's path is $\frac{-\hat{i} + 5\hat{j}}{\sqrt{26}}$, the curvature of its path has a value of $\frac{1}{6}$, and its acceleration vector is $(\hat{i} + \hat{j})\sqrt{26}$. What is its speed and at what rate is it speeding up or slowing down at P ?
6. An object is traveling in the counterclockwise direction around the ellipse $4x^2 + 9y^2 = 36$. When it reaches the point $x = 0, y = -2$ meters, its acceleration vector is $3i + 5j$ m/s². What is its speed and at what rate is it speeding up or slowing down at that time?
7. The total force on an object with mass 8 kg and having a velocity vector $4\hat{j}$ m/s is $\hat{i} - 3\hat{j}$ Newtons. Find the curvature of its path at that moment.
8. A coin with mass ρ kg will not slide off a spinning turntable provided the force on it is no greater than $\frac{2g}{9}$ N. How far from the center of a turntable spinning at 45 revolutions per minute can a coin be without sliding off?
9. What is the maximum magnitude of the force required to cause an object weighing 1 kg to move at the constant speed of 1 m/s along the parabola $y = x^2$?
10. A particle moves along the parabola $y = x^2$ in such a way that the horizontal component of its velocity is always 1 m/s. At the point (1,1), find the tangential and normal components of the acceleration vector.